Combining signal processing and computer modelling and simulation in cardiac arrhythmia studies

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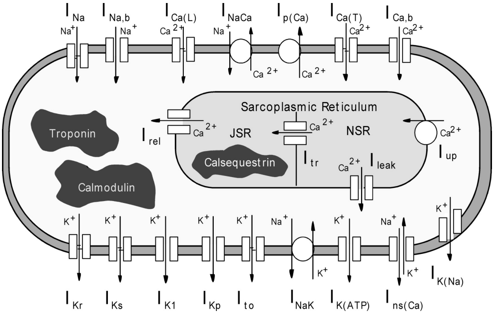
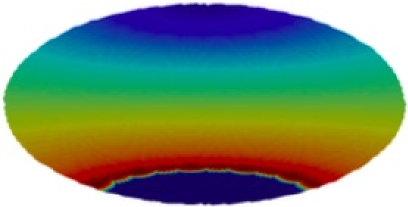
Bioelectricity

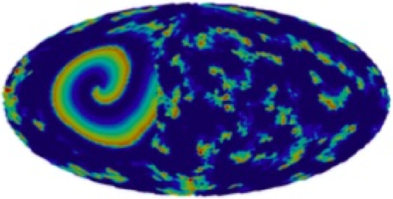


Kléber and Rudy, Physiological Reviews 2004: 84:431-488

Cardiac arrhythmias

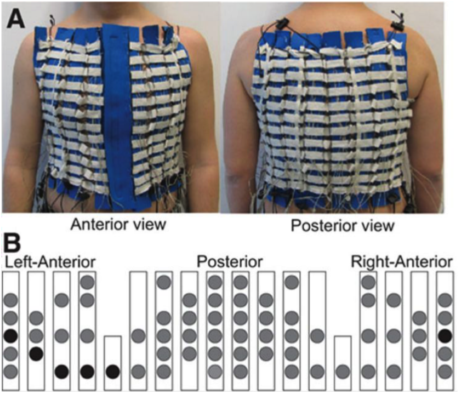


Regular rhythm



Irregular rhythm

Cardiac signals



Guillem et al. Circ Arrhythm Electrophysiol. 2013;6:294-301.

Open question

Best electrode locations? How many electrodes?

Meds $\longrightarrow$ Maths

Cardiac rhythm/arrhythmia $\longrightarrow$ Set of spatiotemporal dynamics.

Diagnosis $\longrightarrow$ Characterisation/identification of spatiotemporal dynamics.

Prevention and treatment $\longrightarrow$ Control of spatiotemporal dynamics.

Problem formulation

- $\mathbf{x}(t)$ corresponds to a cardiac rhythm.
- $\mathbf{x}(t)$ produces cardiac signals $\mathbf{y}(t)$.
- Given cardiac signals $\mathbf{y}(t)$, what can we say about the underlying cardiac rhythm $\mathbf{x}(t)$?

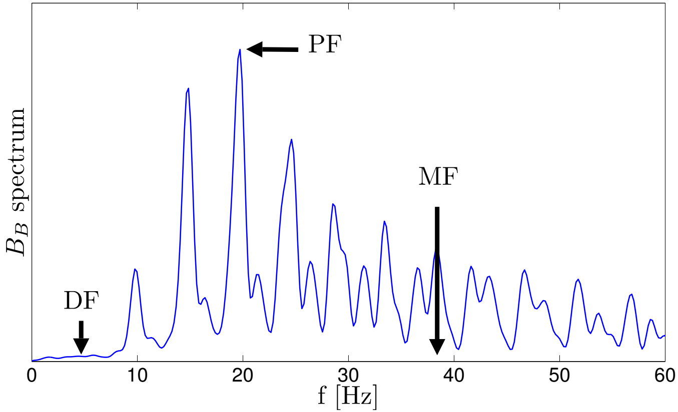
Signal processing approaches

Approach: Characterise an underlying cardiac rhythm by using a collection of signal parameters $\boldsymbol{\theta}$ extracted from recorded cardiac signals $\mathbf{y}(t)$.

Examples:

- Temporal/morphological parameters.
- Spectral parameters.
- Complexity parameters.

Spatiotemporal organisation and spectral analysis



Requena-Carrión et al. Biomed Signal Proces. 2013: 8:744-739

Bioelectric modelling: The forward problem

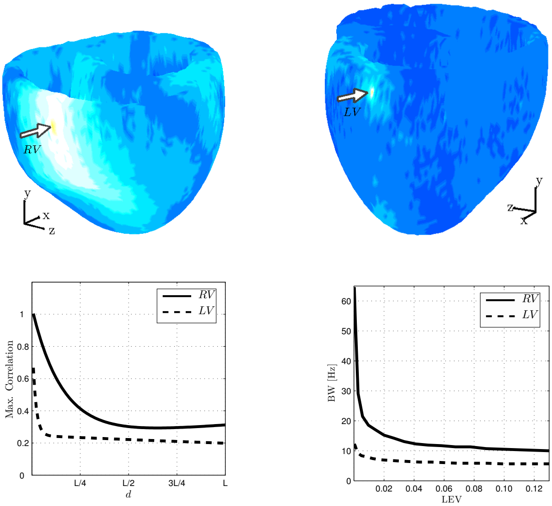
$\mathbf{x}$: Cardiac potential

$\mathbf{y}$: Thorso potential

Forward problem:

$$\mathbf{x} \longrightarrow \mathbf{y} = \mathbf{A}\mathbf{x}$$

Spectral distortion



Requena-Carrión et al. Biomed Signal Proces. 2013: 8:935-944

Leveraging bioelectric models: Inverse solutions

Forward problem:

$$\mathbf{x} \longrightarrow \mathbf{y} = \mathbf{A}\mathbf{x}$$

Inverse problem:

$$\mathbf{y} \longrightarrow \mathbf{x}?$$

Optimisation

Fewer observations $\mathbf{y}$ than unknowns $\mathbf{x}$: **Undetermined problem**.

Usual strategy: **Regularisation**

$$\min_{\mathbf{x}}\{\|\mathbf{y}-\mathbf{A}\mathbf{x}\|+\lambda F(\mathbf{x})\}$$

For instance:

$$\min_{\mathbf{x}}\{\|\mathbf{y}-\mathbf{A}\mathbf{x}\|+\lambda\|\mathbf{L}\mathbf{x}\|\}\Rightarrow\mathbf{x}=\left(\mathbf{A}^T\mathbf{A}+\lambda\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{A}^T\mathbf{y}$$

Is this good enough? Physiologically meaningful? Accurate? Numerically stable?

Exploiting physiological priors

In addition to the linear relationship between $\mathbf{x}$ and $\mathbf{y}$, we know:

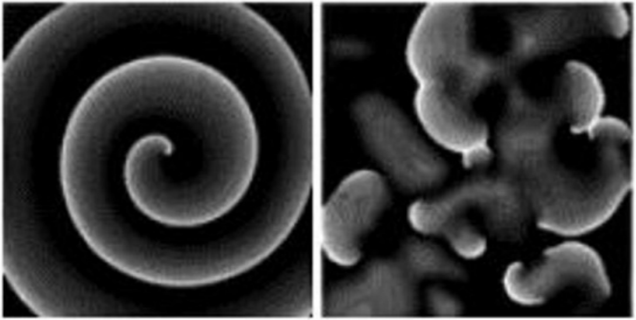
- **Physiological range** of values for $\mathbf{x}$.
- **Spatial correlation**: $\mathbf{x}(s, t_0) \simeq \mathbf{x}(s + \Delta s, t_0)$.
- **Temporal correlation**: $\mathbf{x}(s_0, t) \simeq \mathbf{x}(s_0, t + \Delta t)$.

New (physiologically more meaningful) optimisation problems:

$$\min_{\mathbf{x}} \{ \|\mathbf{y} - \mathbf{A}\mathbf{x}\| + \lambda_1 \|\mathbf{L}\mathbf{x}\| + \lambda_2 \|\mathbf{x}(t + \Delta t) - \rho \mathbf{x}(t) + \theta\| + \lambda_3 \|\theta\| \}$$
$$\text{s.t} \quad -90 \text{ mV} < \mathbf{x} < 20 \text{ mV}$$

Is this good enough? Physiologically meaningful? Accurate? Numerically stable?

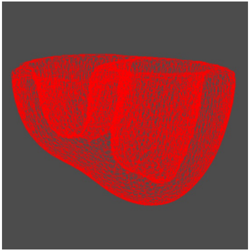
Computer simulation of cardiac dynamics



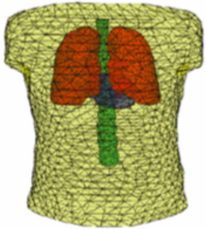
Spiral wave   Multiple wavelet

Muramatsu and Takayama, J Cerebrovasc Dis & Stroke. 2014;1(4): 1018.

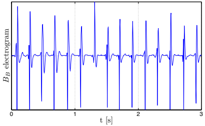
Computer simulation of cardiac signals



Cardiac state $\mathbf{s}$, potential $\mathbf{x}$



Transfer matrix $\mathbf{A}$



Cardiac signal $\mathbf{y}$, parameter θ

State-space approach

State-space model

$$\begin{aligned}\mathbf{s}(t+\Delta t) &= F(\mathbf{s}(t)) + \Delta\mathbf{s}(t)\\ \mathbf{x}(t) &= G(\mathbf{s}(t))\end{aligned}$$

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{A}\mathbf{x}(t)\\ \theta &= \Theta(\mathbf{y}(t))\end{aligned}$$

Observation errors

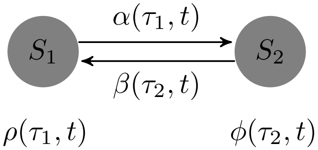
$$\Delta\mathbf{y}(t)$$
$$\Delta\theta$$

State correction

$$\Delta\mathbf{s}(t) = H(\mathbf{s}(t), \Delta\mathbf{y}(t), \Delta\theta)$$

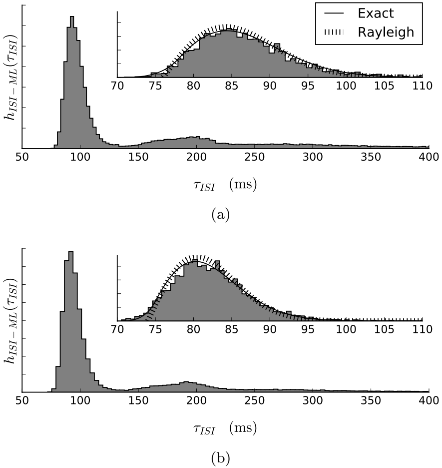
By incorporating modelling and simulation, cardiac signal analysis can be seen as a (cardiac state) **tracking** problem.

Sufficient cardiac model



Requena-Carrion, Proyecto Fin de Carrera, Universidad Carlos III de Madrid, Spain (2003).

Neuronal inter-spike interval distribution



Requena-Carrion and Requena-Carrion, Phys Rev E 2016; 93:042418

Open question

Best sensing locations to characterise arrhythmias?

But...

What is an arrhythmia?

Rephrasing the open question

In a state-space approach:

- Cardiac arrhythmias are **groups of trajectories** in the state space.
- Identifying arrhythmias means **distinguishing trajectories**.
- **Sensing partitions the space** where trajectories live.

This view leads to the problems:

- Sensor arrangement → partition of the space of trajectories?
- Set of trajectories → sensor arrangement?

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