“Spectral Mixture Gaussian Processes are Probabilistic Filter Banks”:
Unifying Probabilistic Models for Time-Frequency Analysis

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"Overview"

- Gaussian processes (GPs) are a probabilistic machine learning approach that allow us to learn distributions over functions. Useful tools for regression, interpolation, extrapolation and noise removal.
- A spectral mixture GP (1) models the covariance as a sum of quasi-periodic components [1].
- The probabilistic phase vocoder (PPV) (2) is a signal processing method that allows us to fit a filter bank to a signal by leveraging uncertainty [2].
- GPs have equivalent representations as stochastic differential equations (SDEs) [3].
- By formulating its SDE representation, we show that the Matérn spectral mixture GP (1) is exactly equivalent to the PPV (2).
- We leverage the best of both worlds for inference on audio signals: fast frequency-domain optimisation, interpretability, easy to switch out kernels, guaranteed stationarity.

"Results"

- This generative model can handle missing data synthesis, denoising, source separation.
- Swapping the kernel for a higher-order Matérn-$\nu$ allows instantaneous frequency to be correlated through time & improves missing data synthesis:

\begin{align*}
  f(t) &\sim \text{GP}\left(0, \sum_{d=1}^{D} C^{(d)}_{q q}(t,t') \right) \\
  y_k &= f(t_k) + \sigma_k z_k
\end{align*}  \tag{1}

- A GP model whose kernel is a sum of quasi-periodic covariance functions:

\[ C^{(d)}_{q q}(t,t') = \sigma_q^2 \cos(\omega_d (t-t')) \exp\left(-\frac{|t-t'|}{\ell_d}\right) \]

- $\ell_d$ = lengthscale,
- $\omega_d$ = frequency,
- $\sigma_q^2$ = variance.

- Fast inference via Kalman filtering
- Fast frequency domain parameter learning
- Interpreting the model is challenging
- Changing the model is easy
- Changing the model is hard

"References"


Fig. 1: Missing data synthesis example with the probabilistic filter bank using 3 different Matérn-$\nu$ kernels: the Matérn-$1/2$ (PPV), the Matérn-$3/2$ and the Matérn-$5/2$. 

Code and resources available: https://github.com/wil-j-wil/unifying-prob-time-freq